How long does it take to melt ice off an automobile with an infrared space heater?

Student Name:	**David DeWolf**
Student Number:	**28594639**
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Abstract

While working at a car cleaning service job, management would continually complain that the workers were taken too long to clear the ice of the vehicles and would say that the 30,000 BTU radiant heater in the garage could melt the ice faster than a worker could. On average, it would take a worker no longer than 15 minutes to clear off a foot of ice that accumulated on the hood of the vehicle. I wanted to show management that I could produce a mathematical model to show how long it would take the heater to melt the ice off the vehicles at various uniform ice temperatures at several ice thicknesses. I found that the time required to melt just 1-inch of snow at 23 ° F would take 23.1 minutes and if the ice were as thick as a foot, it would take 14.2 hours to completely melt the ice. To clear a foot of ice off an automobile at 23 ° F with the heater alone takes 57 times longer than doing it by a worker. Now if the ice were even colder at -4 ° F, it would take 29.8 minutes to melt 1-inch and it takes nearly 19.5 hours to completely melt a foot of ice. A foot of ice at this temperature would take the heater 78 times longer than by a worker. As the ice thickness increased at a given ice temperature, the time required to melt the ice exponentially increased

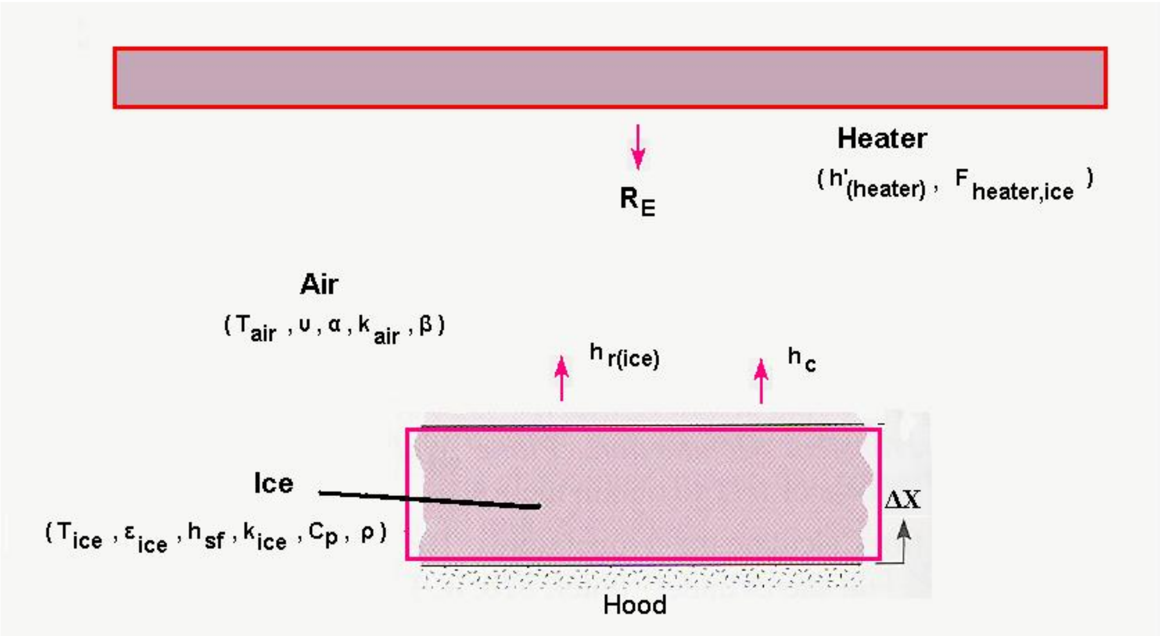
Introduction

In order to completely melt ice, the ice has to go through two phases. The first phase is the sensible heat phase. In this phase you're putting energy into the system and the temperature steadily rises until it reaches 0 ° C or 32 ° F. Then when the ice hits the freezing temperature, it is called the insensible heat phase. This phase doesn't seem to make sense because you keep putting energy into the system and the temperature hasn't moved a degree. You keep putting heat in the system because of the latent heat of diffusion is absorbing that energy to reach the next desired stage. These two phases is why it takes time to melt ice because of the energy needed to put in the system.

Project Description

I will investigate the heat transfer of ice from a 30,000 BTU heater in an enclosed area of a garage. I will need to find the fraction of radiation energy leaving the heater and intercepted by the ice or otherwise known as the view factor. I will need to calculate the net heat transfer coefficient in order to calculate the Biot number. Once I have the Biot number, I can determine whether or not the lumped capacitance general equation will work in order to find the transient time for the first phase. If the lumped capacitance method is in valid, I will solve the first phase by using the implicit method to find the corresponding time. Once this phase is calculated, I can find the time required in the insensible heat phase and sum the two times to find the total time. Both these methods will be done for ice at 32 ° F, 23 ° F, and -4 ° F with ice thickness ranging from 1-inch to a foot.

Project sketch



Theory / Analysis

What is Calculated

I will calculate the time required to melt ice off an automobile at 32 ° F, 23 ° F, and -4 ° F with ice thickness ranging from 1-inch to a foot of ice by using a 30,000 BTU heater.

Energy Balance

1st Phase:

Calculate Biot number

$$Bi = \frac{h_{net}\Delta x}{k_{ice}}$$ No Units (if Bi < 0.1 , use lump capacitance)

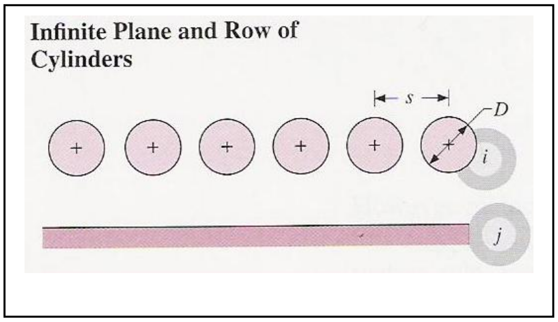
Δx = increment (m)

h_{net} is the total heat transfer. $h_{net} = h'_{(heater)} - h_{r(ice)} - h_c$ $\left(\frac{W}{m^2K}\right)$

h'(heater) = $\dfrac{R_E F_{heater,ice}}{(T_{air} - T_{ice}) * (A_{ice})}$ $\left(\dfrac{W}{m^2 K}\right)$

R_E = energy of heater. (W)

$F_{heater,ice}$ = View factor No Units



$F_{ij} = 1 - \sqrt{1 - \left(\dfrac{D}{s}\right)^2} + \left(\dfrac{D}{s}\right)\tan^{-1}\left(\sqrt{\dfrac{s^2 - D^2}{D^2}}\right)$

For n rows of in-line pipes:

$F_{1\text{-}n\ rows} = 1 - (1 - F_{ij})^n$

n = 2 rows

T_{air} = temperature of air (K)

T_{ice} = temperature of ice (K)

A_{ice} = area of snow (m^2)

$h_{r(ice)} = \varepsilon_{ice}\sigma(T_{ice} + T_{air})(T_{ice}^2 + T_{air}^2)$ $\left(\dfrac{W}{m^2 K}\right)$

ε_{ice} = emissivity of ice No Units

σ = Stefan-Boltzmann constant = 5.67 X 10^{-8} $\left(\dfrac{W}{m^2 K^4}\right)$

$h_c = \dfrac{(Nu_L)(k_{air})}{L}$ $\left(\dfrac{W}{m^2 K}\right)$

Cold Plate Facing Up

$Nu_L = 0.15\sqrt[3]{Ra_L}$ No Units for $10^7 \le Ra_L \le 10^{11}$

$Ra_L = \dfrac{g\beta(T_{air} - T_{ice})L^3}{\alpha\nu}$ No Units

g = gravitational force (9.81 m/s^2)

β = expansion coefficient (K^{-1})

α = thermal diffusivity (m^2/s)

υ = kinematic viscosity (m^2/s)

L = characteristic length = $\left(\frac{A_{ice}}{P_{ice}}\right)$ (m)

A_{ice} = area of ice (m^2)

P_{ice} = perimeter of ice (m)

Calculate Fourier Number

Fo = $\frac{\alpha_{ice}\Delta t}{(\Delta x)^2}$ No Units

Δt = time (s)

α_{ice} = thermal diffusivity of ice = $\frac{k_{ice}}{\rho C_p}$ (m^2/s)

k_{ice} = thermal conductivity of ice $\left(\frac{W}{mK}\right)$

ρ = density $\left(\frac{kg}{m^3}\right)$

C_p = specific heat $\left(\frac{J}{kgK}\right)$

Δx = increment (m)

Use Implicit Form

Find the time it takes hood surface to reach 0 ° C or 32 ° F.

Surface Nodes

$(1 + 2Fo + 2FoBi)T_o^{P+1} - 2FoT_1^{P+1} = 2FoBiT_\infty + T_o^P$

Interior Nodes

$(1 + 2Fo)T_m^{P+1} – Fo(T_{m-1}^{P+1} + T_{m+1}^{P+1} = T_m^P$

2nd Phase:

Calculate time during insensible heat phase

$$(h_{net})A_{ice}(T_{air} - T_{ice}) = \frac{M_{ice}h_{sf}}{t}$$

M_{ice} = mass of ice = volume*density (kg)

h_{sf} = latent heat of fusion (J / kg)

t = time (what is going to be calculated) (s)

Assumptions

- Transient problem (solving the time required to melt ice)
- Use implicit method to solve for time in the sensible heat phase
- Choose small size increments for smaller gradients in the ice
- Hood is an insulated surface
- The View factor is assumed for an isothermal piece of ice
- Infinite plane to row of parallel cylinders

Hand Calculation

First I will need to calculate Biot number to see if the lumped capacitance method is valid. I will start out doing calculations of ice at 23° F which is 1 inch thick.

Bi = $\dfrac{h_{net}\Delta x}{k_{ice}}$

$h_{net} = h'_{(heater)} - h_{r(ice)} - h_c$

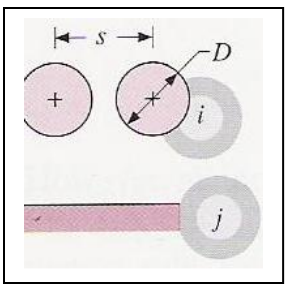
$h'_{(heater)} = \dfrac{R_E F_{heater,ice}}{(T_{air} - T_{ice}) * (A_{ice})}$

$R_E = \left(\dfrac{30{,}000 BTU}{h}\right)\left(\dfrac{1kW}{3412 BTU/h}\right) = 8.79 kW$

T_{air} = 68 ° F or 20 ° C or 293 K

T_{ice} = 23 ° F or -5 ° C or 268K

View Factor can be obtained using formula from Table # 4 in the Tables section



$F_{ij} = 1 - \sqrt{1 - \left(\dfrac{D}{s}\right)^2} + \left(\dfrac{D}{s}\right)\tan^{-1}\left(\sqrt{\dfrac{s^2 - D^2}{D^2}}\right)$

For n rows of in-line pipes:

$F_{1\text{-}n\ rows} = 1 - (1 - F_{ij})^n$

n = 2 rows , s = 5 inches , and D = 3 inches

$F_{heater,ice} = 0.94$

Hood is 48 inches X 36 inches or 1.2 m X 0.914 m

$A_{ice} = 1.11\ m^2$

$h'_{(heater)} = \dfrac{8.79E^3 * 0.94}{(293K - 268K) * (1.11 m^2)} = 297.8 \dfrac{W}{m^2 K}$

$h_{r(ice)} = \varepsilon_{ice}\sigma(T_{ice} + T_{air})(T_{ice}^2 + T_{air}^2)$

$h_{r(ice)} = 0.96 * \left(5.67E^{-8}\dfrac{W}{m^2K^4}\right)(268K + 293K)(268K^2 + 293K^2) = 4.81 \dfrac{W}{m^2 K}$

$h_c = \frac{(Nu_L)(k_{air})}{L}$

Cold Plate Facing Up

$Ra_L = \frac{g\beta(T_{air} - T_{ice})L^3}{\alpha\nu}$

Properties of Air at 293 K can be obtained from Table # 3 and then interpolating the correct value.

$g = 9.81\ m/s^2$; $\beta = \frac{1}{293K} = 0.0034K^{-1}$; $\alpha = 2.16E^{-5}\ m/s^2$; $\upsilon = 1.53E^{-5}\ m/s^2$;

$k = 0.257$ W/mK ; $L = \left(\frac{A_{ice}}{P_{ice}}\right)$; $P_{ice} = 2(\ 48\ in + 1\ in\ ) = 98\ in$ or $2.48\ m$

$$Ra_L = \frac{(9.81m/s^2)(0.0034K^{-1})(0.47m)^3(293K - 268K)}{(1.53E^{-5}m/s^2)(2.16E^{-5}m/s^2)} = 2.28E^8$$

This is free convection with a cold plate facing up, this Rayleigh number falls between

$10^7 \le Ra_L \le 10^{11}$; $Nu_L = 0.15\sqrt[3]{Ra_L}$

$$Nu_L = 0.15\sqrt[3]{2.28E^8} = 91.6$$

$$h_c = \frac{(91.6)(0.0257W/mK)}{(0.47m)} = 5.26\frac{W}{m^2K}$$

Overall net heat transfer coefficient is

$h_{net} = h'_{(heater)} - h_{r(ice)} - h_c$

$$h_{net} = (286.8 - 4.81 - 5.26)W/m^2K = 286.79\frac{W}{m^2K}$$

1 inch of ice = 0.0254 m

$$Bi = \frac{(286.79W/m^2K)(0.0254m)}{(1.9175W/mK)} = 3.8$$

Since Bi is > 0.1 , Lumped capacitance cannot be used

I will solve transient problem using implicit method

I will arbitrarily choose 8 equal distances to keep the gradients small within the ice

$$\text{Let } \Delta x = \frac{0.0254m}{8} = 0.003175m$$

Let $\Delta t =$ 30 seconds

Bi = $\dfrac{(286.79W/m^2K)(0.003175m)}{(1.9175W/mK)} = 0.47$

ρ and C_p are obtained from Table #1 and Table # 2 respectively

$\rho = 917.$ kg/m^3 and $C_p = 2016.25$ J/kgK

Fo = $\dfrac{\alpha_{ice}\Delta t}{(\Delta x)^2} = \dfrac{k_{ice}\Delta t}{\rho C_p \Delta x^2} = \dfrac{(1.9175W/mK)(30\sec)}{(2016.25J/kgK)(917.4kg/m^3)(0.003175m)^3} = 3.08$

Implicit Method

Find the time it takes hood surface or T_0 to reach 0 ° C or 32 ° F.

Surface Nodes has the form of

$(1 + 2Fo + 2FoBi)T_o^{P+1} - 2FoT_1^{P+1} = 2FoBiT_\infty + T_o^P$

Interior Nodes have the form of

$(1 + 2Fo)T_m^{P+1} – Fo(T_{m-1}^{P+1} + T_{m+1}^{P+1} = T_m^P$

Using these two main equations, write down corresponding equations for each segment

$(1 + 2Fo)T_0^{p+1} – 2Fo(T_1^{p+1}) = T_0^p$

$(1 + 2Fo)T_1^{p+1} – Fo(T_0^{p+1} + T_2^{p+1}) = T_1^p$

$(1 + 2Fo)T_2^{p+1} – Fo(T_1^{p+1} + T_3^{p+1}) = T_2^p$

$(1 + 2Fo)T_3^{p+1} – Fo(T_2^{p+1} + T_4^{p+1}) = T_3^p$

$(1 + 2Fo)T_4^{p+1} – Fo(T_3^{p+1} + T_5^{p+1}) = T_4^p$

$(1 + 2Fo)T_5^{p+1} – Fo(T_4^{p+1} + T_6^{p+1}) = T_5^p$

$(1 + 2Fo)T_6^{p+1} – Fo(T_5^{p+1} + T_7^{p+1}) = T_6^p$

$(1 + 2Fo)T_7^{p+1} – Fo(T_6^{p+1} + T_8^{p+1}) = T_7^p$

$(1 + 2Fo + 2FoBi)T_8^{p+1} – 2FoT_7^{p+1} = 2FoBiT_{air} + T_8^p$

Substituting corresponding values

$$\begin{bmatrix} 7.16 & -6.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3.08 & 7.16 & -3.08 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.08 & 7.16 & -3.08 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.08 & 7.16 & -3.08 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.08 & 7.16 & -3.08 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.08 & 7.16 & -3.08 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.08 & 7.16 & -3.08 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.08 & 7.16 & -3.08 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6.17 & 10.09 \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{bmatrix} = \begin{bmatrix} T_0^P \\ T_1^P \\ T_2^P \\ T_3^P \\ T_4^P \\ T_5^P \\ T_6^P \\ T_7^P \\ 58.6+T_8^P \end{bmatrix}$$

"A" "X" "C"

Now if we perform matrix inversion and create a chart, every time a value is computed above zero, it must be manually changed to 0.

$X = A^{-1}C$

Implicit Finite-Difference Solution for Δt = 30 seconds										
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**
0	0	-5	-5	-5	-5	-5	-5	-5	-5	-5
1	30	-4.752	-4.711	-4.578	-4.307	-3.811	-2.93	-1.379	1.32	6.14

We see that the values at the top surface T_8 and T_7 are greater than zero so manually change its value to 0 and proceed with next inversion with new corrected values. Final results can be seen for each case in the Appendix section. For this particular case, it takes T_0 to reach 0 ° C when p = 9 or t = 270 seconds.

Now that the time for the implicit method was found, we can calculate the time during the insensible heat phase.

$$(h_{net})(A_{ice})(T_{air} - T_{ice}) = \frac{M_{ice} h_{sf}}{t}$$

M_{ice} = volume*density ; h_{sf} = latent heat of fusion, can be calculated on Table # 1

h_{sf} = 343.84 kJ/kg

Now solve for the corresponding time

$$t = \frac{(\rho * A_{ice} * thickness) h_{sf}}{(h_{net})(A_{ice})(T_{air} - T_{ice})} = \frac{(917.47 kg/m^3)(0.0254m)(343.845E^3 J/kg)}{(286.79 W/m^2K)(293K - 268K)} = 1{,}117.6 \sec$$

Total time is to add part 1 and part 2.

Total Time = 270 sec + 1,117.6 sec = 1,387.6 seconds or 23.1 minutes

For 1 inch of ice at 23 ° F, it would take the 30,000 BTU heater a total time of 23.1 minutes to completely melt. This following approach was done for all calculations and put on spreadsheet which can be seen in the Appendix section. Results can be seen for ice at 23 ° F, 32 ° F, and -4 ° F at various ice thicknesses.

Discussion

The total time required to melt ice was broken up into two stages if the uniform temperature of the ice was below the freezing point or otherwise it only takes 1 step to calculate the time. For example, ice at uniform temperature of 23 ° F took time in the sensible and the insensible heat phases. Looking at Figure # 2 in the Figures section, the sensible heat time rose exponentially as the ice thickness increased. Ice that was a foot thick took 10.5 hours to reach the insensible heat phase. Once the ice reached a uniform freezing point temperature, then the time it took during this phase linearly increased with increasing ice thickness. Ice that was thick as a foot took an extra 3.7 hours to completely melt the ice. With both heat phases combined, it takes a little bit over 14 hours to completely melt a foot of ice. The total time to melt ice at -4 ° F also acted in the same manner. A foot of ice at this temperature took even longer for each heat phase as the sensible heat stage took 15.3 hours and the insensible heat phase took an extra 4.1 hours for a total of 19.4 hours to completely melt ice off the automobile. The results for uniform ice can be seen in Figure # 3. Ice that was already at the freezing point or at 32 ° F only has to go through the insensible heat phase. Time linearly increases as the ice thickness increases. The time to completely melt ice at 1 inch takes about 18 minutes and ice that is a foot thick takes 3.6 hours or about 12% longer. These results can be seen on Figure # 1. The total times for all three cases can be seen on Figure # 4. As the temperature gets colder and colder the total time required to melt the ice increases as expected.

Conclusion

It is very clear that management is wrong to think that the radiant heater in the garage could do a quicker job than a worker. What a worker does in about 15 minutes would take the radiant heater alone about 57% longer for a foot of ice at a uniform temperature of 23 ° F. It would take almost 78% longer for the heater to melt that same foot of ice at -4 ° F. The results show that ice takes a lot of energy input to completely melt. The sensible heat phase is the major contributor in the time it takes to melt the ice as it acts exponentially when the ice thickness increases at a given temperature below the freezing point. The insensible heat phase is just an extra added on time that acts linearly as the thickness of ice increases to make the time even greater. Maybe a radiant heater could do a faster job if a larger heater was at this work place but that's not the case. I have concluded that the workers are doing an excellent job compared to the time it takes the 30,000 BTU heater in the garage to melt the ice alone.

References

1. Incropera F.P, and DeWitt D. (2002) Introduction to Heat Transfer (4th Edition). John Wiley and Sons, New York
2. Howell, J.R. (1981) *A Catalog of Radiation Configuration Factors*. McGraw-Hill, New York
3. Cengel A.Y, and Boles A.M. (2002) Thermodynamics: An Engineering Approach(4th Edition). McGraw-Hill, New York

Tables

Saturated ice—water vapor

Temp., T °C	Sat. press., P_{sat} kPa	Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
		Sat. ice, $v_i \times 10^3$	Sat. vapor, v_g	Sat. ice, u_i	Subl., u_{ig}	Sat. vapor, u_g	Sat. ice, h_i	Subl., h_{ig}	Sat. vapor, h_g	Sat. ice, s_i	Subl., s_{ig}	Sat. vapor, s_g
0.01	0.6113	1.0908	206.1	−333.40	2708.7	2375.3	−333.40	2834.8	2501.4	−1.221	10.378	9.156
0	0.6108	1.0908	206.3	−333.43	2708.8	2375.3	−333.43	2834.8	2501.3	−1.221	10.378	9.157
−2	0.5176	1.0904	241.7	−337.62	2710.2	2372.6	−337.62	2835.3	2497.7	−1.237	10.456	9.219
−4	0.4375	1.0901	283.8	−341.78	2711.6	2369.8	−341.78	2835.7	2494.0	−1.253	10.536	9.283
−6	0.3689	1.0898	334.2	−345.91	2712.9	2367.0	−345.91	2836.2	2490.3	−1.268	10.616	9.348
−8	0.3102	1.0894	394.4	−350.02	2714.2	2364.2	−350.02	2836.6	2486.6	−1.284	10.698	9.414
−10	0.2602	1.0891	466.7	−354.09	2715.5	2361.4	−354.09	2837.0	2482.9	−1.299	10.781	9.481
−12	0.2176	1.0888	553.7	−358.14	2716.8	2358.7	−358.14	2837.3	2479.2	−1.315	10.865	9.550
−14	0.1815	1.0884	658.8	−362.15	2718.0	2355.9	−362.15	2837.6	2475.5	−1.331	10.950	9.619
−16	0.1510	1.0881	786.0	−366.14	2719.2	2353.1	−366.14	2837.9	2471.8	−1.346	11.036	9.690
−18	0.1252	1.0878	940.5	−370.10	2720.4	2350.3	−370.10	2838.2	2468.1	−1.362	11.123	9.762
−20	0.1035	1.0874	1128.6	−374.03	2721.6	2347.5	−374.03	2838.4	2464.3	−1.377	11.212	9.835
−22	0.0853	1.0871	1358.4	−377.93	2722.7	2344.7	−377.93	2838.6	2460.6	−1.393	11.302	9.909
−24	0.0701	1.0868	1640.1	−381.80	2723.7	2342.0	−381.80	2838.7	2456.9	−1.408	11.394	9.985
−26	0.0574	1.0864	1986.4	−385.64	2724.8	2339.2	−385.64	2838.9	2453.2	−1.424	11.486	10.062
−28	0.0469	1.0861	2413.7	−389.45	2725.8	2336.4	−389.45	2839.0	2449.5	−1.439	11.580	10.141
−30	0.0381	1.0858	2943	−393.23	2726.8	2333.6	−393.23	2839.0	2445.8	−1.455	11.676	10.221
−32	0.0309	1.0854	3600	−396.98	2727.8	2330.8	−396.98	2839.1	2442.1	−1.471	11.773	10.303
−34	0.0250	1.0851	4419	−400.71	2728.7	2328.0	−400.71	2839.1	2438.4	−1.486	11.872	10.386
−36	0.0201	1.0848	5444	−404.40	2729.6	2325.2	−404.40	2839.1	2434.7	−1.501	11.972	10.470
−38	0.0161	1.0844	6731	−408.06	2730.5	2322.4	−408.06	2839.0	2430.9	−1.517	12.073	10.556
−40	0.0129	1.0841	8354	−411.70	2731.3	2319.6	−411.70	2839.9	2427.2	−1.532	12.176	10.644

Table # 1

Thermophysical Properties of Common Materials

Description/ Composition	Temperature (K)	Density, ρ (kg/m^3)	Thermal Conductivity, k (W/m · K)	Specific Heat, c_p (J/kg · K)
Ice	273	920	1.88	2040
	253	—	2.03	1945

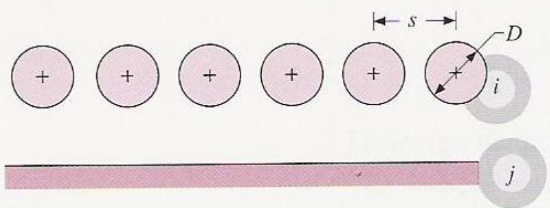
Table # 2

Thermophysical Properties of Gas at Atmospheric Pressure

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707

Table # 3

Infinite Plane and Row of Cylinders



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right) \tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

Table # 4

Figures

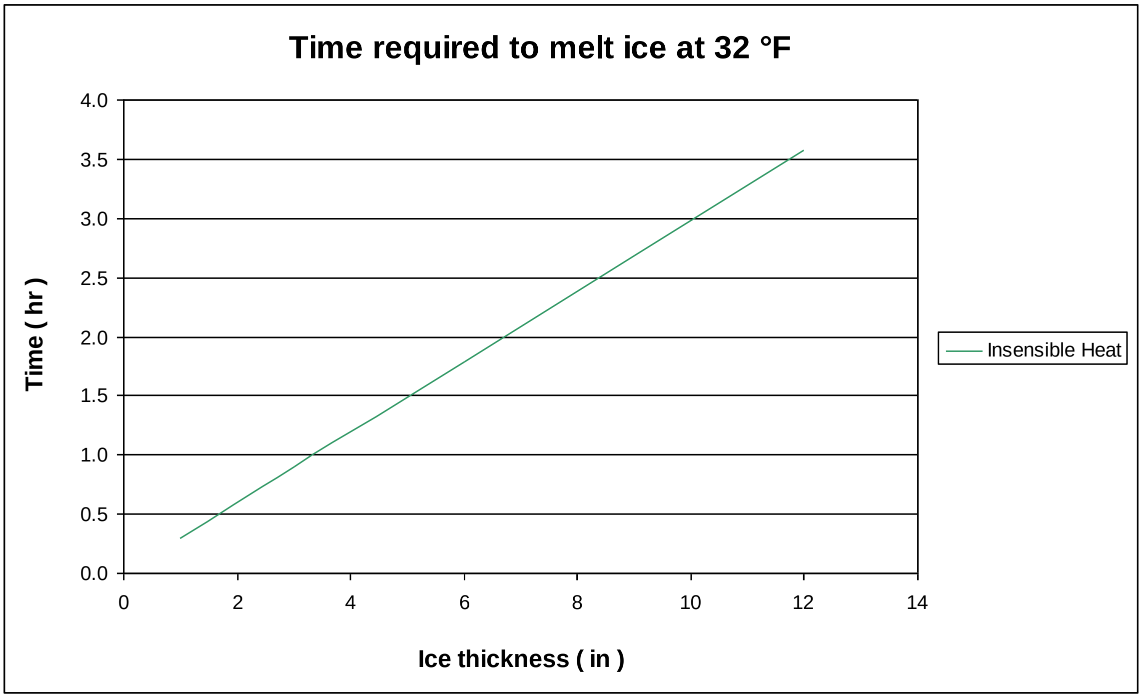


Figure # 1

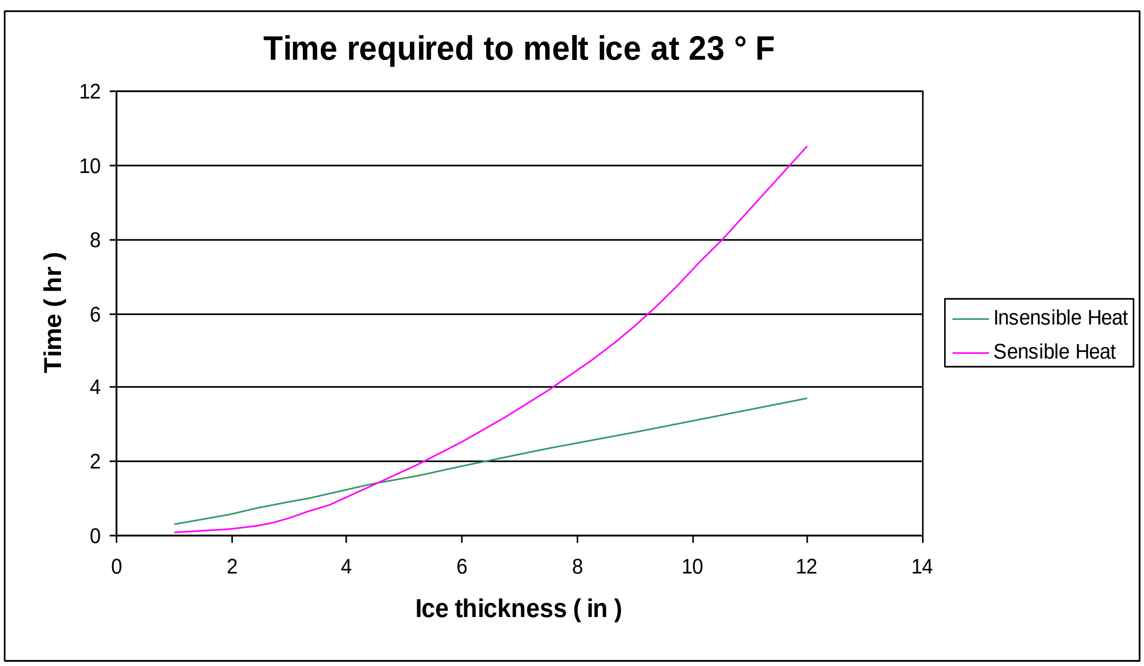


Figure # 2

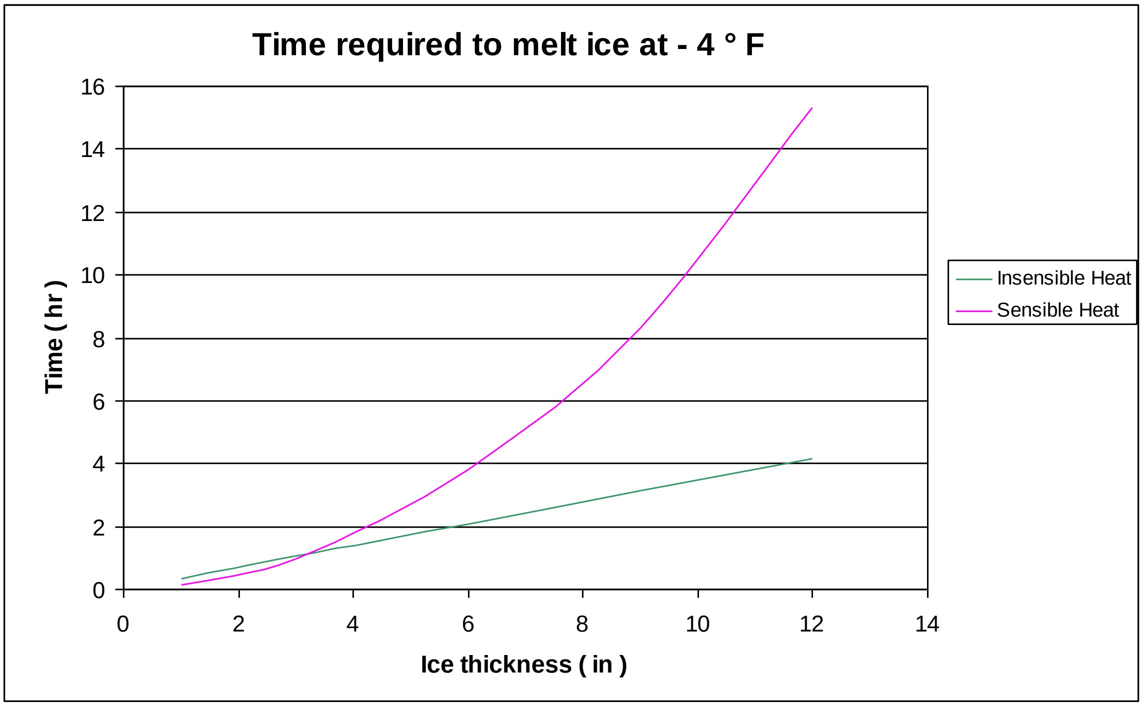


Figure # 3

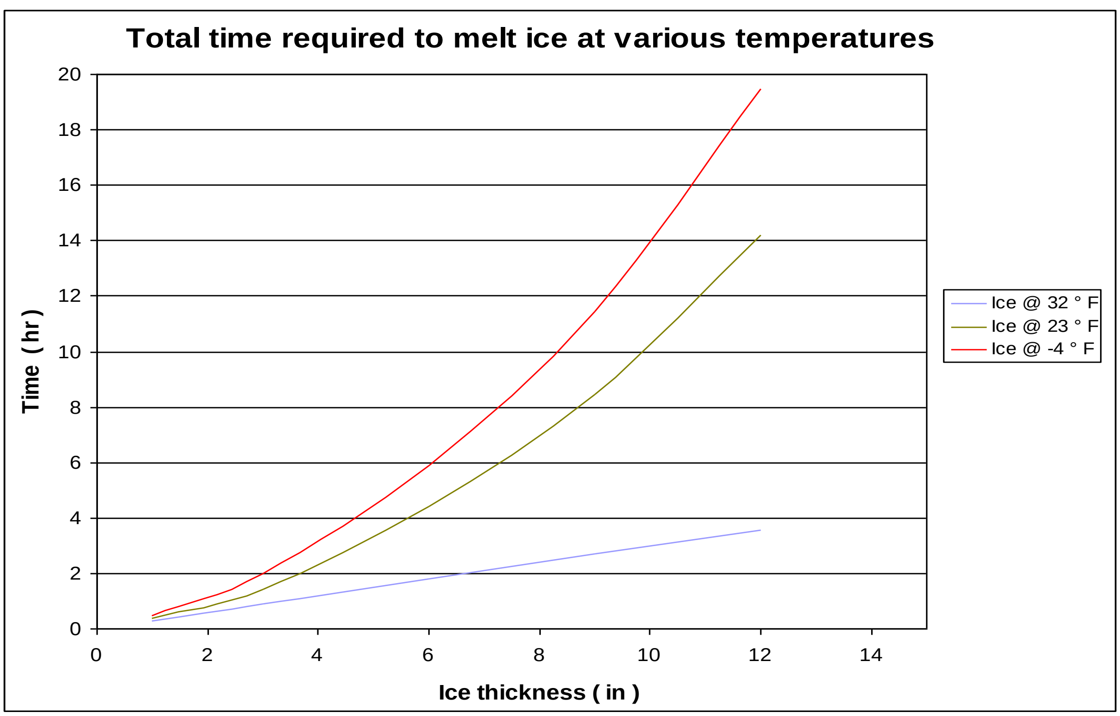


Figure # 4

Appendix

Conversion Factors

1 kW = 3412 Btu/h

1 in = 2.54 cm

1 W = 1 J/s

1 hour = 60 minutes

1 minute = 60 seconds

$T(K) = T(°C) + 273$

$T(°F) = 1.8\,T(°C) + 32$

Hood = 1.219 X 0.914 m

Area$_{(ice)}$ = 1.1143488 m^2

Ice thickness				**Perimeter = 2*(W + t)**			
1 in	=	0.0254	m	98 in	=	2.4888	m
3 in	=	0.0762	m	102 in	=	2.5904	m
6 in	=	0.1524	m	108 in	=	2.7428	m
9 in	=	0.2286	m	114 in	=	2.8952	m
12 in	=	0.3048	m	120 in	=	3.0476	m

Temperature of Air

20 C or 293 K

Temperature of Ice

0 C	or	273	K
-5 C	or	268	K
-20 C	or	253	K

Properties of Ice @ 0 C			**Properties of Ice @ -5 C**			**Properties of Ice @ -20 C**		
ε	0.96		ε	0.96		ε	0.96	
h_{sf} @ 0 C	333.43	kJ/kg	h_{sf} @ -5 C	343.845	kJ/kg	h_{sf} @ -20 C	374.03	kJ/kg
ρ @ 0 C	916.76	kg/m^3	ρ @ -5 C	917.47	kg/m^3	ρ @ -20 C	919.62	kg/m^3
k @ 0 C	1.88	W/mK	k @ -5 C	1.9175	W/mK	k @ -20 C	2.03	W/mK
C_p @ 0 C	2040	J/kgK	C_p @ -5 C	2016.25	J/kgK	C_p @ -20 C	1945	J/kgK
$hr_{(ice)}$ @ 0 C	4.9410075	W/m^2K	$hr_{(ice)}$ @ -5 C	4.81475823	W/m^2K	$hr_{(ice)}$ @ -20 C	4.453760578	W/m^2K

Energy of Heater Output

30000 BTU = 8.7925 kW

View Factor 0.9406479

$h'_{(heater)}$ @ 0 C	371.09763	W/m^2K	$h'_{(heater)}$ @ -5 C	296.878101	W/m^2K	$h'_{(heater)}$ @ -20 C	185.548813	W/m^2K

Cold Plate Facing Up

Air Properties @ 293K

υ	1.53E-05	m^2/s
α	2.16E-05	m^2/s
k	0.02574	W/mK
β	0.003413	K^{-1}
g	9.81	m/s^2

Charchterisic Length

$L = A_{(ice)}/P$

0.44774542	m
0.43018406	m
0.40628146	m
0.38489527	m
0.36564799	m

Ice at 0 C

Ra_L #	Nusselt #	h_c	
1.82E+08	85.079447	4.891	W/m^2K
1.62E+08	81.742483	4.891	W/m^2K
1.36E+08	77.200572	4.891	W/m^2K
1.16E+08	73.136822	4.891	W/m^2K
9.94E+07	69.479501	4.891	W/m^2K

Ice at - 5 C

Ra_L #	Nusselt #	h_c	
2.28E+08	91.649056	5.26872	W/m^2K
2.02E+08	88.0544204	5.26872	W/m^2K
1.70E+08	83.1617948	5.26872	W/m^2K
1.45E+08	78.7842535	5.26872	W/m^2K
1.24E+08	74.8445238	5.26872	W/m^2K

Ice at - 20 C

Ra_L #	Nusselt #	h_c	
3.65E+08	107.19339	6.1623	W/m^2K
3.24E+08	102.98907	6.1623	W/m^2K
2.73E+08	97.266625	6.1623	W/m^2K
2.32E+08	92.146622	6.1623	W/m^2K
1.99E+08	87.538686	6.1623	W/m^2K

Ice Thickness @ 0 C		$h_{net} = h'_{(heater)} - hr_{(ice)} - h_c$		Biot #
0.0254	m	361.27	W/m^2K	4.880928432
0.0762	m	361.27	W/m^2K	14.6427853
0.1524	m	361.27	W/m^2K	29.28557059
0.2286	m	361.27	W/m^2K	43.92835589
0.3048	m	361.27	W/m^2K	58.57114119

Ice Thickness @ -5 C		$h_{net} = h'_{(heater)} - hr_{(ice)} - h_c$		Biot #
0.0254	m	286.79	W/m^2K	3.799000432
0.0762	m	286.79	W/m^2K	11.3970013
0.1524	m	286.79	W/m^2K	22.79400259
0.2286	m	286.79	W/m^2K	34.19100389
0.3048	m	286.79	W/m^2K	45.58800519

Ice Thickness @ -20 C		$h_{net} = h'_{(heater)} - hr_{(ice)} - h_c$		Biot #
0.0254	m	174.93	W/m^2K	2.188813297
0.0762	m	174.93	W/m^2K	6.566439892
0.1524	m	174.93	W/m^2K	13.13287978
0.2286	m	174.93	W/m^2K	19.69931968
0.3048	m	174.93	W/m^2K	26.26575957

***** Since Bi > 0.1   Can't Use Lumped Capacitance Method ******

PHASE # 1 (Sensible Heat)

Since Bi > 0.1 Can't Use Lumped Capacitance Method

Use Implicit Method to find the time for insulated side to reach 0 C

Since the Ice is at 0 C for varoius thicknesses, not necessary to do sensible heat phase stage

Ice Thickness @ -5 C		**Time**				
0.0254	m	270	s	=	0.075	hours
0.0762	m	1800	s	=	0.5	hours
0.1524	m	9180	s	=	2.55	hours
0.2286	m	20400	s	=	5.66666667	hours
0.3048	m	37800	s	=	10.5	hours

Ice Thickness @ -20 C		**Time**				
0.0254	m	540	s	=	0.15	hours
0.0762	m	3500	s	=	0.97222222	hours
0.1524	m	13680	s	=	3.8	hours
0.2286	m	30000	s	=	8.33333333	hours
0.3048	m	55200	s	=	15.3333333	hours

PHASE # 2 (Insensible Heat)

$$(h_{net})Area_{ice}(T_{air} - T_{ice}) = \frac{Mass * h_{sf}}{time}$$

Ice Thickness @ 0 C		**Time**				
0.0254	m	1074.576842	s	=	0.29849357	hours
0.0762	m	3223.730526	s	=	0.8954807	hours
0.1524	m	6447.461052	s	=	1.7909614	hours
0.2286	m	9671.191578	s	=	2.6864421	hours
0.3048	m	12894.9221	s	=	3.58192281	hours

Ice Thickness @ -5 C		**Time**				
0.0254	m	1117.576587	s	=	0.31043794	hours
0.0762	m	3352.72976	s	=	0.93131382	hours
0.1524	m	6705.459521	s	=	1.86262764	hours
0.2286	m	10058.18928	s	=	2.79394147	hours
0.3048	m	13410.91904	s	=	3.72525529	hours

Ice Thickness @ -20 C		**Time**				
0.0254	m	1248.583325	s	=	0.3468287	hours
0.0762	m	3745.749975	s	=	1.0404861	hours
0.1524	m	7491.49995	s	=	2.08097221	hours
0.2286	m	11237.24993	s	=	3.12145831	hours
0.3048	m	14982.9999	s	=	4.16194442	hours

Total Time (Add Phase # 1 and Phase # 2)

Ice Thickness @ 0 C		Time				
0.0254	m	1074.576842	s	=	0.29849357	hours
0.0762	m	3223.730526	s	=	0.8954807	hours
0.1524	m	6447.461052	s	=	1.7909614	hours
0.2286	m	9671.191578	s	=	2.6864421	hours
0.3048	m	12894.9221	s	=	3.58192281	hours

Ice Thickness @ -5 C		Time				
0.0254	m	1387.576587	s	=	0.38543794	hours
0.0762	m	5152.72976	s	=	1.43131382	hours
0.1524	m	15885.45952	s	=	4.41262764	hours
0.2286	m	30458.18928	s	=	8.46060813	hours
0.3048	m	51210.91904	s	=	14.2252553	hours

Ice Thickness @ -20 C		Time				
0.0254	m	1788.583325	s	=	0.4968287	hours
0.0762	m	7245.749975	s	=	2.01270833	hours
0.1524	m	21171.49995	s	=	5.88097221	hours
0.2286	m	41237.24993	s	=	11.4547916	hours
0.3048	m	70182.9999	s	=	19.4952778	hours

Ice Thickness @ -5 C = 0.0254 m **Phase #1**

Let Δx = 0.003175 m
Let Δt = 30 sec

$$Bi = \frac{h_{net}\Delta x}{k_{ice}} = 0.4749 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 3.0848$$

Implicit Finite-Difference Solution for Δt = 30 seconds										
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**
0	0	-5	-5	-5	-5	-5	-5	-5	-5	-5
1	30	-4.7516	-4.7114	-4.5776	-4.3068	-3.8113	-2.9305	-1.3789	0	0
2	60	-4.1693	-4.0749	-3.7742	-3.213	-2.2972	-0.8907	0	0	0
3	90	-3.3756	-3.247	-2.85	-2.1533	-1.1132	0	0	0	0
4	120	-2.5297	-2.3926	-1.9785	-1.2819	-0.3028	0	0	0	0
5	150	-1.7408	-1.6129	-1.2323	-0.6098	0	0	0	0	0
6	180	-1.0628	-0.9529	-0.629	-0.1096	0	0	0	0	0
7	210	-0.5129	-0.4238	-0.1631	0	0	0	0	0	0
8	240	-0.1031	-0.0367	0	0	0	0	0	0	0
9	270	0	0	0	0	0	0	0	0	0

Ice Thickness @ -5 C = 0.0762 m **Phase #1**

Let Δx = 0.00635 m
Let Δt = 300 sec

$$Bi = \frac{h_{net}\Delta x}{k_{ice}} = 0.9497$$

$$fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 7.7121$$

Implicit Finite-Difference Solution for Δt = 300 seconds														
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**	**T_9**	**T_{10}**	**T_{11}**	**T_{12}**
0	0	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
1	300	-4.5094	-4.4775	-4.378	-4.1978	-3.9135	-3.4884	-2.8673	-1.9697	-0.6791	0	0	0	0
2	600	-3.5039	-3.4387	-3.2388	-2.8912	-2.3741	-1.6575	-0.7034	0	0	0	0	0	0
3	900	-2.317	-2.2401	-2.0077	-1.6157	-1.0583	-0.3302	0	0	0	0	0	0	0
4	1200	-1.2244	-1.1535	-0.9418	-0.5919	-0.1092	0	0	0	0	0	0	0	0
5	1500	-0.3538	-0.2974	-0.13	0	0	0	0	0	0	0	0	0	0
6	1800	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -5 C = 0.1542 m **Phase#1**

Let Δx = 0.0101 m
Let Δt = 540 sec

$$Bi = \frac{h_{net}\Delta x}{k_{ice}} = 1.6473 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 4.6140$$

Implicit Finite-Difference Solution for Δt = 540 seconds																
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**	**T_9**	**T_{10}**	**T_{11}**	**T_{12}**	**T_{13}**	**T_{14}**
0	0	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
1	540	-4.939	-4.933	-4.9117	-4.8714	-4.8032	-4.6924	-4.5149	-4.2323	-3.783	-3.0707	-1.9399	-0.1458	0	0	0
2	1080	-4.753	-4.733	-4.669	-4.5527	-4.3675	-4.0877	-3.677	-3.0846	-2.243	-1.0686	0	0	0	0	0
3	1620	-4.43	-4.395	-4.2867	-4.0957	-3.8057	-3.3938	-2.8316	-2.0862	-1.124	0	0	0	0	0	0
4	2160	-4.006	-3.96	-3.8195	-3.578	-3.2242	-2.7445	-2.124	-1.3501	-0.417	0	0	0	0	0	0
5	2700	-3.53	-3.478	-3.3226	-3.059	-2.6829	-2.1895	-1.5759	-0.8434	-0.001	0	0	0	0	0	0
6	3240	-3.043	-2.99	-2.8315	-2.5665	-2.1947	-1.7171	-1.1371	-0.4621	0	0	0	0	0	0	0
7	3780	-2.573	-2.522	-2.3696	-2.1171	-1.7672	-1.3246	-0.7969	-0.1956	0	0	0	0	0	0	0
8	4320	-2.136	-2.088	-1.9467	-1.7136	-1.393	-0.9914	-0.5175	0	0	0	0	0	0	0	0
9	4860	-1.737	-1.694	-1.5652	-1.3539	-1.0646	-0.7042	-0.2816	0	0	0	0	0	0	0	0
10	5400	-1.381	-1.343	-1.228	-1.0403	-0.7845	-0.4681	-0.1005	0	0	0	0	0	0	0	0
11	5940	-1.068	-1.034	-0.9332	-0.7684	-0.5448	-0.2691	0	0	0	0	0	0	0	0	0
12	6480	-0.796	-0.766	-0.6787	-0.536	-0.3429	-0.106	0	0	0	0	0	0	0	0	0
13	7020	-0.563	-0.538	-0.4636	-0.3424	-0.1793	0	0	0	0	0	0	0	0	0	0
14	7560	-0.368	-0.347	-0.2846	-0.1833	-0.0475	0	0	0	0	0	0	0	0	0	0
15	8100	-0.21	-0.193	-0.1418	-0.0602	0	0	0	0	0	0	0	0	0	0	0
16	8640	-0.086	-0.073	-0.0338	0	0	0	0	0	0	0	0	0	0	0	0
17	9180	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -5 C = 0.2286 m **Phase #1**

Let Δx = 0.0143 m
x
Let Δt = 1200 sec

$$Bi = \frac{h_{net}\Delta x}{k_{ice}} = 2.1369 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 6.0935$$

Implicit Finite-Difference Solution for Δt = 1200 seconds																		
p	t (s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}
0	0	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
1	1200	-4.933	-4.9275	-4.9101	-4.8779	-4.8258	-4.745	-4.6224	-4.4378	-4.1609	-3.7463	-3.126	-2.1982	-0.8105	0	0	0	0
2	2400	-4.73	-4.7134	-4.6616	-4.569	-4.4257	-4.2167	-3.9211	-3.5104	-2.9474	-2.1853	-1.1671	0	0	0	0	0	0
3	3600	-4.3829	-4.3544	-4.267	-4.1149	-3.8882	-3.5734	-3.1529	-2.6064	-1.9116	-1.0468	0	0	0	0	0	0	0
4	4800	-3.9324	-3.8955	-3.7832	-3.5915	-3.3139	-2.9421	-2.4667	-1.8787	-1.1712	-0.3423	0	0	0	0	0	0	0
5	6000	-3.4332	-3.3922	-3.2686	-3.0606	-2.7655	-2.3804	-1.903	-1.3332	-0.6739	0	0	0	0	0	0	0	0
6	7200	-2.9282	-2.8867	-2.7624	-2.5549	-2.2644	-1.8917	-1.4389	-0.9098	-0.3113	0	0	0	0	0	0	0	0
7	8400	-2.4458	-2.4062	-2.2878	-2.0914	-1.8191	-1.4736	-1.0595	-0.5832	-0.0532	0	0	0	0	0	0	0	0
8	9600	-2.0003	-1.9637	-1.8546	-1.6743	-1.4256	-1.1123	-0.7397	-0.3146	0	0	0	0	0	0	0	0	0
9	10800	-1.5989	-1.566	-1.4678	-1.3061	-1.084	-0.8058	-0.4774	-0.1058	0	0	0	0	0	0	0	0	0
10	12000	-1.2411	-1.2117	-1.1242	-0.9803	-0.783	-0.5362	-0.2452	0	0	0	0	0	0	0	0	0	0
11	13200	-0.9298	-0.9043	-0.8283	-0.7037	-0.5338	-0.323	-0.0772	0	0	0	0	0	0	0	0	0	0
12	14400	-0.664	-0.6422	-0.5773	-0.4713	-0.3271	-0.1491	0	0	0	0	0	0	0	0	0	0	0
13	15600	-0.4411	-0.4228	-0.3685	-0.2799	-0.1599	-0.0125	0	0	0	0	0	0	0	0	0	0	0
14	16800	-0.2582	-0.2432	-0.1988	-0.1264	-0.0289	0	0	0	0	0	0	0	0	0	0	0	0
15	18000	-0.1148	-0.1031	-0.0683	-0.0121	0	0	0	0	0	0	0	0	0	0	0	0	0
16	19200	-0.0101	-0.0015	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	20400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -5 C   = 0.3048 m   **Phase #1**

Let Δx = 0.017 m
Let Δt = 1800 sec

$$Bi = \frac{h_{net}\Delta x}{k_{ice}} = 2.1369 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 6.0935$$

Implicit Finite-Difference Solution for Δt = 1800 seconds																				
p	t (s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}
0	0	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
1	1800	-4.961	-4.958	-4.9487	-4.931	-4.904	-4.8608	-4.7966	-4.7013	-4.56	-4.3512	-4.0426	-3.5869	-2.914	-1.92	-0.4539	0	0	0	0
2	3600	-4.833	-4.823	-4.792	-4.737	-4.653	-4.5295	-4.3555	-4.1137	-3.7816	-3.3298	-2.7211	-1.9094	-0.8398	0	0	0	0	0	0
3	5400	-4.595	-4.577	-4.5208	-4.423	-4.277	-4.0735	-3.7997	-3.4405	-2.9779	-2.3917	-1.6614	-0.7682	0	0	0	0	0	0	0
4	7200	-4.265	-4.24	-4.1626	-4.03	-3.838	-3.5777	-3.2414	-2.8193	-2.3018	-1.6803	-0.9495	-0.1094	0	0	0	0	0	0	0
5	9000	-3.876	-3.846	-3.7552	-3.602	-3.383	-3.0946	-2.7317	-2.2904	-1.7678	-1.1632	-0.4792	0	0	0	0	0	0	0	0
6	10800	-3.46	-3.428	-3.3321	-3.171	-2.944	-2.6486	-2.2851	-1.853	-1.3536	-0.7906	-0.1704	0	0	0	0	0	0	0	0
7	12600	-3.044	-3.012	-2.9157	-2.756	-2.532	-2.2448	-1.8957	-1.4867	-1.0215	-0.5052	0	0	0	0	0	0	0	0	0
8	14400	-2.643	-2.612	-2.5193	-2.366	-2.153	-1.8817	-1.5546	-1.175	-0.7476	-0.278	0	0	0	0	0	0	0	0	0
9	16200	-2.266	-2.237	-2.1507	-2.008	-1.809	-1.5584	-1.2577	-0.9113	-0.5245	-0.1033	0	0	0	0	0	0	0	0	0
10	18000	-1.919	-1.893	-1.8129	-1.681	-1.5	-1.2703	-0.9968	-0.6831	-0.3344	0	0	0	0	0	0	0	0	0	0
11	19800	-1.603	-1.579	-1.5064	-1.387	-1.222	-1.0147	-0.7679	-0.486	-0.1738	0	0	0	0	0	0	0	0	0	0
12	21600	-1.319	-1.297	-1.2317	-1.124	-0.976	-0.7909	-0.571	-0.3208	-0.0452	0	0	0	0	0	0	0	0	0	0
13	23400	-1.065	-1.045	-0.9871	-0.891	-0.76	-0.5949	-0.3999	-0.1787	0	0	0	0	0	0	0	0	0	0	0
14	25200	-0.84	-0.823	-0.7717	-0.687	-0.571	-0.4266	-0.256	-0.0632	0	0	0	0	0	0	0	0	0	0	0
15	27000	-0.644	-0.629	-0.5842	-0.51	-0.41	-0.2838	-0.1361	0	0	0	0	0	0	0	0	0	0	0	0
16	28800	-0.475	-0.462	-0.4228	-0.359	-0.273	-0.1646	-0.0384	0	0	0	0	0	0	0	0	0	0	0	0
17	30600	-0.331	-0.32	-0.2866	-0.233	-0.159	-0.0687	0	0	0	0	0	0	0	0	0	0	0	0	0
18	32400	-0.211	-0.202	-0.1743	-0.13	-0.069	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	34200	-0.114	-0.106	-0.0843	-0.048	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	36000	-0.039	-0.033	-0.0165	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	37800	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -20 C = 0.0254 m **Phase #1**

Let Δx = 0.003175 m
Let Δt = 60 sec

$$Bi = \frac{h_{net}\,\Delta x}{k_{ice}} = 0.2736 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 6.7551$$

Implicit Finite-Difference Solution for Δt = 60 seconds										
p	**t (s)**	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
0	0	-20	-20	-20	-20	-20	-20	-20	-20	-20
1	60	-18.4561	-18.3418	-17.9821	-17.3236	-16.2689	-14.6619	-12.2647	-8.7223	-3.5104
2	120	-15.651	-15.4434	-14.8067	-13.7	-12.0568	-9.79	-6.8021	-3.0055	0
3	180	-12.3381	-12.0929	-11.3517	-10.0991	-8.3133	-5.9734	-3.0685	0	0
4	240	-9.0461	-8.8024	-8.0716	-6.8553	-5.1587	-2.9952	-0.3908	0	0
5	300	-6.07	-5.8497	-5.1923	-4.1086	-2.6184	-0.752	0	0	0
6	360	-3.5542	-3.368	-2.8144	-1.9088	-0.6776	0	0	0	0
7	420	-1.5511	-1.4028	-0.9636	-0.2505	0	0	0	0	0
8	480	-0.8334	-0.6156	0	0	0	0	0	0	0
9	540	0	0	0	0	0	0	0	0	0

Ice Thickness @ -20 C = 0.0762 m **Phase #1**

Let Δx = 0.00635 m
Let Δt = 500 sec

$$Bi = \frac{h_{net}\,\Delta x}{k_{ice}} = 0.5472 \qquad fo = \frac{\alpha\Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 14.0731$$

Implicit Finite-Difference Solution for Δt = 500 seconds														
p	t (s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
0	0	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20
1	500	-17.792	-17.7131	-17.4721	-17.0515	-16.4214	-15.537	-14.336	-12.732	-10.611	-7.823	-4.17	0	0
2	1000	-13.956	-13.8202	-13.4073	-12.7056	-11.695	-10.349	-8.6336	-6.5134	-3.9514	-0.916	0	0	0
3	1500	-9.7536	-9.6043	-9.1554	-8.4044	-7.3477	-5.9822	-4.3063	-2.323	-0.0419	0	0	0	0
4	2000	-5.9445	-5.8092	-5.4042	-4.7326	-3.8002	-2.6156	-1.1918	0	0	0	0	0	0
5	2500	-2.8431	-2.7329	-2.4041	-1.8622	-1.1162	-0.1796	0	0	0	0	0	0	0
6	3000	-0.5308	-0.4487	-0.2042	0	0	0	0	0	0	0	0	0	0
7	3500	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -20 C = 0.1542 m **Phase #1**

Let Δx = 0.0101 m
Let Δt = 720 sec

$$Bi = \frac{h_{net}\Delta x}{k_{ice}} = 0.9380 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 6.8958$$

Implicit Finite-Difference Solution for Δt = 720 seconds																
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**	**T_9**	**T_{10}**	**T_{11}**	**T_{12}**	**T_{13}**	**T_{14}**
0	0	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20
1	720	-19.72	-19.7	-19.632	-19.515	-19.327	-19.041	-18.616	-17.99	-17.07	-15.731	-13.77	-10.906	-6.723	-0.615	0
2	1440	-18.94	-18.88	-18.708	-18.399	-17.929	-17.256	-16.324	-15.06	-13.37	-11.146	-8.2552	-4.5648	0	0	0
3	2160	-17.68	-17.59	-17.307	-16.824	-16.113	-15.138	-13.856	-12.217	-10.16	-7.6477	-4.6235	-1.0726	0	0	0
4	2880	-16.08	-15.97	-15.619	-15.025	-14.169	-13.032	-11.59	-9.8183	-7.699	-5.2225	-2.3941	0	0	0	0
5	3600	-14.33	-14.21	-13.823	-13.18	-12.269	-11.082	-9.6126	-7.8565	-5.816	-3.5023	-0.9392	0	0	0	0
6	4320	-12.55	-12.43	-12.039	-11.393	-10.488	-9.3255	-7.9079	-6.2431	-4.344	-2.2321	0	0	0	0	0
7	5040	-10.84	-10.71	-10.337	-9.7166	-8.8531	-7.7526	-6.4239	-4.88	-3.138	-1.222	0	0	0	0	0
8	5760	-9.223	-9.106	-8.757	-8.1785	-7.3769	-6.3612	-5.1437	-3.7406	-2.172	-0.4639	0	0	0	0	0
9	6480	-7.743	-7.636	-7.315	-6.7853	-6.0535	-5.1299	-4.0276	-2.7636	-1.358	0	0	0	0	0	0
10	7200	-6.402	-6.304	-6.0139	-5.5348	-4.8745	-4.0431	-3.0542	-1.9241	-0.672	0	0	0	0	0	0
11	7920	-5.201	-5.114	-4.8539	-4.4259	-3.8372	-3.098	-2.2218	-1.2249	-0.127	0	0	0	0	0	0
12	8640	-4.135	-4.058	-3.8278	-3.4487	-2.9278	-2.2751	-1.5031	-0.6269	0	0	0	0	0	0	0
13	9360	-3.201	-3.134	-2.9317	-2.6	-2.1452	-1.5769	-0.9074	-0.1514	0	0	0	0	0	0	0
14	10080	-2.391	-2.333	-2.1577	-1.8705	-1.4776	-0.9879	-0.4128	0	0	0	0	0	0	0	0
15	10800	-1.7	-1.65	-1.5007	-1.2563	-0.9228	-0.5089	-0.0255	0	0	0	0	0	0	0	0
16	11520	-1.119	-1.077	-0.9518	-0.747	-0.4684	-0.1239	0	0	0	0	0	0	0	0	0
17	12240	-0.646	-0.612	-0.5101	-0.3443	-0.1202	0	0	0	0	0	0	0	0	0	0
18	12960	-0.277	-0.25	-0.1711	-0.0427	0	0	0	0	0	0	0	0	0	0	0
19	13680	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -20 C = 0.2286 m **Phase #1**

Let Δx = 0.0143 m
Let Δt = 1500 sec

$$Bi = \frac{h_{net}\,\Delta x}{k_{ice}} = 1.2312 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 8.3396$$

Implicit Finite-Difference Solution for Δt = 1500 seconds																		
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**	**T_9**	**T_{10}**	**T_{11}**	**T_{12}**	**T_{13}**	**T_{14}**	**T_{15}**	**T_{16}**
0	0	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20
1	1500	-19.749	-19.7339	-19.687	-19.6025	-19.47	-19.2747	-18.992	-18.5885	-18.0158	-17.2051	-16.059	-14.441	-12.156	-8.93	-4.378	0	0
2	3000	-19.0414	-18.999	-18.8684	-18.6397	-18.296	-17.8106	-17.15	-16.2685	-15.1089	-13.6007	-11.66	-9.1924	-6.0951	-2.27	0	0	0
3	4500	-17.8742	-17.8042	-17.5909	-17.2245	-16.688	-15.9595	-15.0087	-13.8012	-12.2977	-10.4572	-8.2398	-5.6122	-2.5553	0	0	0	0
4	6000	-16.3774	-16.2876	-16.016	-15.5556	-14.895	-14.0195	-12.9112	-11.5515	-9.922	-8.0077	-5.7996	-3.2989	-0.5209	0	0	0	0
5	7500	-14.7082	-14.6081	-14.3067	-13.8003	-13.083	-12.1492	-10.9909	-9.6022	-7.9798	-6.1245	-4.0434	-1.7517	0	0	0	0	0
6	9000	-12.9955	-12.8928	-12.5844	-12.0695	-11.347	-10.4165	-9.2781	-7.9343	-6.3905	-4.6561	-2.7457	-0.6797	0	0	0	0	0
7	10500	-11.325	-11.2249	-10.9247	-10.4256	-9.7293	-8.839	-7.7595	-6.498	-5.0643	-3.4715	-1.7367	0	0	0	0	0	0
8	12000	-9.7457	-9.651	-9.3676	-8.8975	-8.2442	-7.4127	-6.4103	-5.2461	-3.9317	-2.4815	-0.9127	0	0	0	0	0	0
9	13500	-8.2842	-8.1966	-7.9345	-7.5007	-6.8993	-6.1367	-5.221	-4.1628	-2.9747	-1.6718	-0.2718	0	0	0	0	0	0
10	15000	-6.9506	-6.8706	-6.6317	-6.2365	-5.6897	-4.9979	-4.1696	-3.2151	-2.1471	-0.9797	0	0	0	0	0	0	0
11	16500	-5.7469	-5.6747	-5.4592	-5.103	-4.6109	-3.9895	-3.2471	-2.3942	-1.4428	-0.4069	0	0	0	0	0	0	0
12	18000	-4.6711	-4.6065	-4.414	-4.096	-3.6574	-3.1044	-2.4453	-1.69	-0.8503	0	0	0	0	0	0	0	0
13	19500	-3.7171	-3.6599	-3.4891	-3.2075	-2.8194	-2.3308	-1.7494	-1.0845	-0.3471	0	0	0	0	0	0	0	0
14	21000	-2.8807	-2.8305	-2.6809	-2.4344	-2.0952	-1.6692	-1.1638	-0.5882	0	0	0	0	0	0	0	0	0
15	22500	-2.1548	-2.1112	-1.9815	-1.7678	-1.4743	-1.1062	-0.6707	-0.176	0	0	0	0	0	0	0	0	0
16	24000	-1.5348	-1.4976	-1.3869	-1.2049	-0.9553	-0.6435	-0.2763	0	0	0	0	0	0	0	0	0	0
17	25500	-1.0154	-0.9842	-0.8915	-0.7394	-0.5315	-0.2728	0	0	0	0	0	0	0	0	0	0	0
18	27000	-0.5904	-0.5649	-0.4891	-0.3651	0	0	0	0	0	0	0	0	0	0	0	0	0
19	28500	-0.2567	-0.2367	-0.1774	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	30000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Ice Thickness @ -20 C = 0.3048 m **Phase #1**

Let Δx = 0.017 m
Let Δt = 2400 sec

$$Bi = \frac{h_{net}\,\Delta x}{k_{ice}} = 1.4592 \qquad fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{k_{ice} * \Delta t}{\rho * C_p * (\Delta x)^2} = 9.4997$$

Implicit Finite-Difference Solution for Δt = 2400 seconds																				
p	**t (s)**	**T_0**	**T_1**	**T_2**	**T_3**	**T_4**	**T_5**	**T_6**	**T_7**	**T_8**	**T_9**	**T_{10}**	**T_{11}**	**T_{12}**	**T_{13}**	**T_{14}**	**T_{15}**	**T_{16}**	**T_{17}**	**T_{18}**
0	0	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20
1	2400	-19.81	-19.8	-19.763	-19.71	-19.62	-19.491	-19.309	-19.055	-18.702	-18.212	-17.53	-16.595	-15.298	-13.51	-11.032	-7.613	-2.8898	0	0
2	4800	-19.23	-19.2	-19.108	-18.95	-18.7	-18.366	-17.91	-17.306	-16.518	-15.5	-14.2	-12.5424	-10.461	-7.871	-4.6876	-0.836	0	0	0
3	7200	-18.25	-18.19	-18.036	-17.77	-17.37	-16.835	-16.139	-15.256	-14.157	-12.81	-11.18	-9.232	-6.9357	-4.268	-1.2214	0	0	0	0
4	9600	-16.94	-16.87	-16.665	-16.31	-15.81	-15.138	-14.289	-13.246	-11.991	-10.509	-8.784	-6.8062	-4.5735	-2.092	0	0	0	0	0
5	12000	-15.45	-15.37	-15.131	-14.73	-14.17	-13.43	-12.513	-11.409	-10.111	-8.6156	-6.921	-5.03	-2.9522	-0.704	0	0	0	0	0
6	14400	-13.88	-13.79	-13.547	-13.13	-12.55	-11.795	-10.869	-9.7701	-8.4987	-7.0575	-5.452	-3.6927	-1.7922	0	0	0	0	0	0
7	16800	-12.32	-12.23	-11.988	-11.58	-11	-10.267	-9.3692	-8.3137	-7.1049	-5.7494	-4.256	-2.637	-0.9068	0	0	0	0	0	0
8	19200	-10.82	-10.74	-10.501	-10.11	-9.56	-8.8606	-8.0129	-7.0225	-5.8961	-4.6424	-3.272	-1.7985	-0.2365	0	0	0	0	0	0
9	21600	-9.406	-9.332	-9.1097	-8.741	-8.229	-7.5757	-6.7877	-5.8707	-4.8325	-3.6823	-2.431	-1.0912	0	0	0	0	0	0	0
10	24000	-8.1	-8.031	-7.8254	-7.485	-7.011	-6.4102	-5.6862	-4.8463	-3.8986	-2.8526	-1.719	-0.5109	0	0	0	0	0	0	0
11	26400	-6.903	-6.84	-6.6521	-6.34	-5.908	-5.3595	-4.7005	-3.9377	-3.0793	-2.1346	-1.114	-0.0304	0	0	0	0	0	0	0
12	28800	-5.816	-5.759	-5.5874	-5.304	-4.912	-4.4147	-3.8181	-3.1286	-2.3539	-1.5029	-0.585	0	0	0	0	0	0	0	0
13	31200	-4.835	-4.784	-4.6294	-4.374	-4.021	-3.5746	-3.0395	-2.4224	-1.7309	-0.9739	-0.161	0	0	0	0	0	0	0	0
14	33600	-3.958	-3.911	-3.7733	-3.545	-3.23	-2.831	-2.3539	-1.8047	-1.1905	-0.5194	0	0	0	0	0	0	0	0	0
15	36000	-3.178	-3.137	-3.0145	-2.812	-2.533	-2.1796	-1.7581	-1.2739	-0.7337	-0.1455	0	0	0	0	0	0	0	0	0
16	38400	-2.492	-2.456	-2.3478	-2.17	-1.924	-1.6145	-1.2454	-0.8222	-0.3515	0	0	0	0	0	0	0	0	0	0
17	40800	-1.894	-1.863	-1.7687	-1.614	-1.4	-1.1319	-0.8126	-0.4477	-0.0434	0	0	0	0	0	0	0	0	0	0
18	43200	-1.379	-1.352	-1.2715	-1.138	-0.955	-0.7251	-0.4522	-0.1413	0	0	0	0	0	0	0	0	0	0	0
19	45600	-0.945	-0.922	-0.8534	-0.741	-0.587	-0.3942	-0.1666	0	0	0	0	0	0	0	0	0	0	0	0
20	48000	-0.585	-0.566	-0.51	-0.418	-0.291	-0.1333	0	0	0	0	0	0	0	0	0	0	0	0	0
21	50400	-0.298	-0.283	-0.238	-0.164	-0.064	0	0		0	0	0	0	0	0	0	0	0	0	0
22	52800	-0.082	-0.07	-0.0363	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	55200	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0