Analysis of a RC and RL Circuit

In this project we will compare the transient analysis of a forced RC and RL circuits by using the computer program PSPISE and will compare those response values with the theoretical values obtained by solving the differential equations involved with each circuit.

Theoretical Analysis of a RC Circuit:

The RC Circuit will contain a 12V dc independent voltage source, a resistor, and a capacitor in series. We can find the step response of a first-order RC circuit by analyzing the circuit shown in Figure 1.

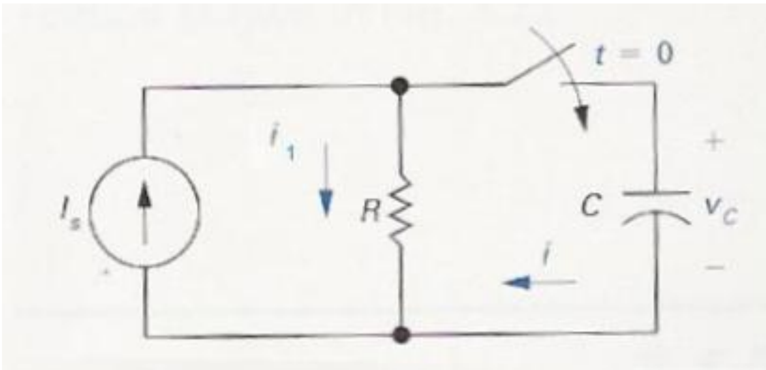

Figure 1

We chose a Norton equivalent for mathematical convenience. If we sum the currents away on the top node in the figure, it generates

$$I_s = i_1 + i_c = \frac{V}{R} + C\frac{dv}{dt}$$

We can divide by C and rearrange which gives

$$\frac{dv}{dt} = -\frac{1}{RC}(V - I_s R)$$

Multiply by dt

$$dv = -\frac{1}{RC}(V - I_s R)dt$$

Rearrange once again

$$\frac{dv}{(V - I_s R)} = -\frac{1}{RC}dt$$

Integrate and put dummy variables in equation

$$\int_{V_o}^{V(t)} \frac{dx}{(x - I_s R)} = -\frac{1}{RC} \int_0^t dy$$

Perform the integration

$$\ln\left(\frac{V_C(t) - I_s R}{V_o - I_s R}\right) = -\frac{t}{RC}$$

Take exponential of both sides and rearrange and the voltage for the capacitor is

$$V_C(t) = I_S R + (V_o - I_S R)\exp\left(-\frac{t}{\tau}\right)$$, where τ = RC equation # 1

The voltage across the resistor is

$$V_R(t) = V_S(t) - V_C(t)$$, where $V_S(t) = I_s R$ equation # 2

Now that the differential equation is solved, the values that our group chose were:

$V_S(t) = 12$ V

V_o = 0V

R = 10kΩ

C = 40μF

Therefore

τ = RC = (10,000Ω)*(40 X 10^{-6} F) = 0.4 seconds

Plot until 5τ = 2 seconds

Use equation # 1 to find the voltage across the capacitor

$$V_C(t) = I_S R + (V_o - I_S R)\exp\left(-\frac{t}{\tau}\right)$$

$V_C(t)$ = 12V – (0 – 12V) exp(-2.5t)

$V_C(t)$ = 12 – 12exp(-2.5t) V

The plot for the voltage vs. time for the capacitor can be seen in Figure 2.

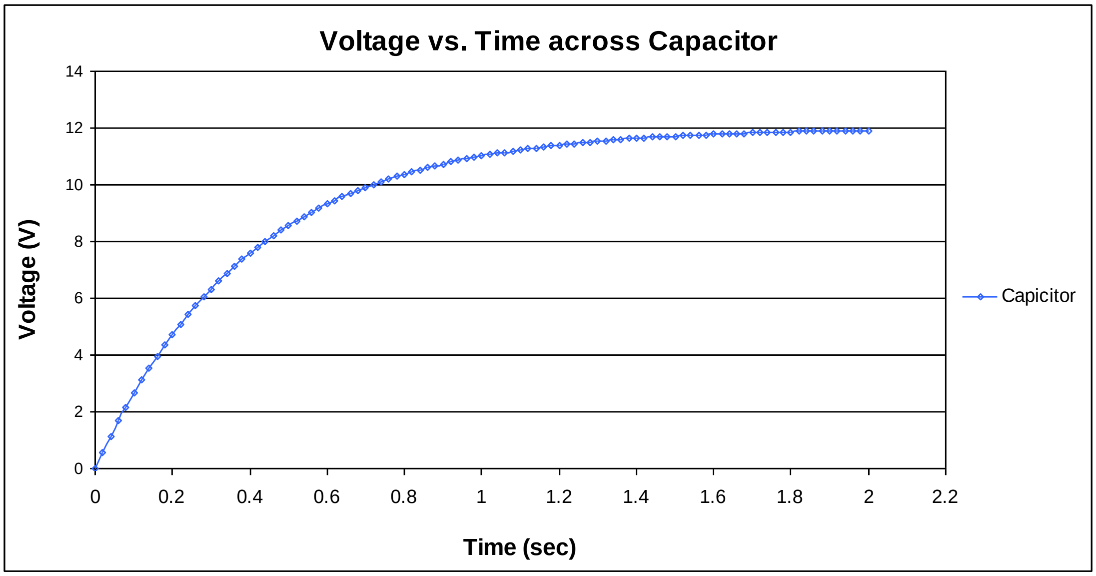

Figure 2

Use equation # 2 to find the voltage across the resistor

$V_R(t) = V_S(t) - V_C(t)$

$V_R(t)$ = 12V – [12 – 12exp(-2.5t) V]

$V_R(t)$ = 12exp(-2.5t) V

The plot for the voltage vs. time across the resistor can be seen in Figure 3.

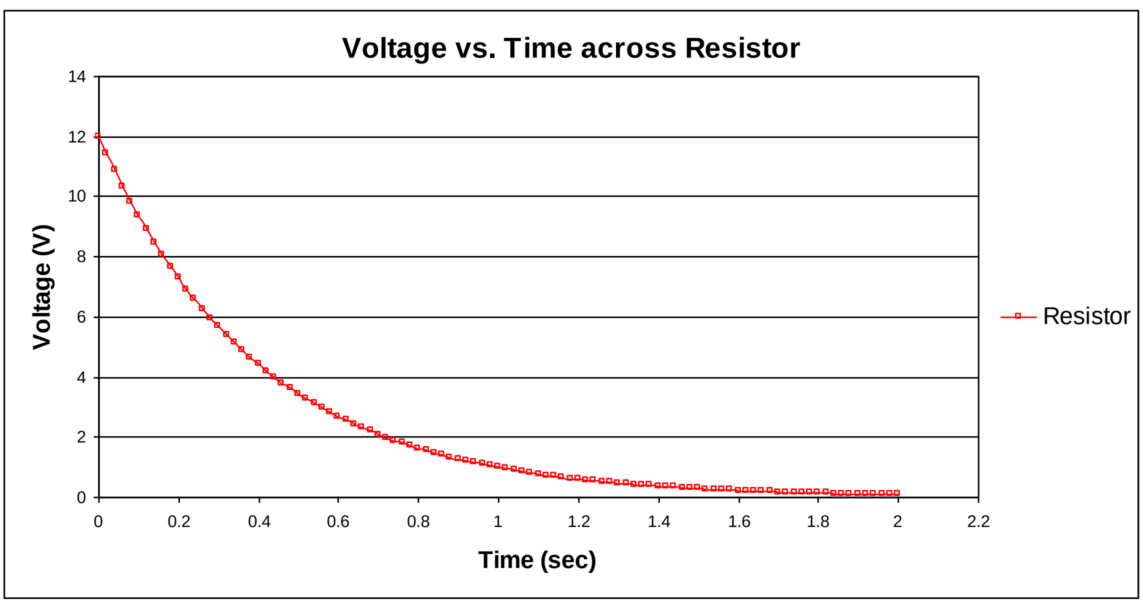

Figure 3

PSPICE Analysis of a RC Circuit:

For this particular part, we used the program PSPICE to perform the transient analysis. Figure 4 shows the circuit layout with the values we arbitrarily chose.

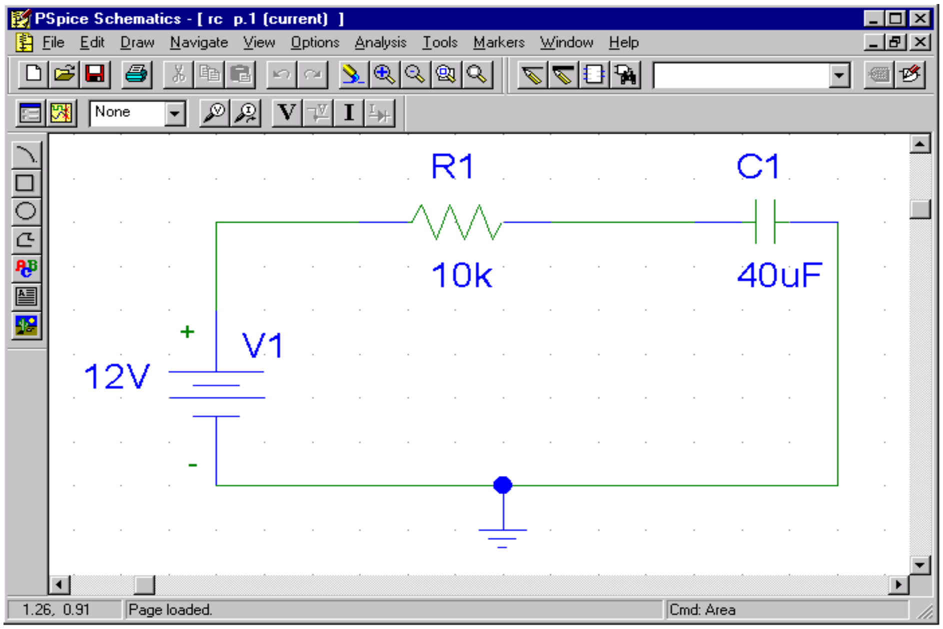

Figure 4

Since our final time will be 2 seconds, under the "Print Step" we set it equal to 0.02 seconds. The plot of voltage vs. time across the capacitor using PSPICE can be seen in Figure 5.

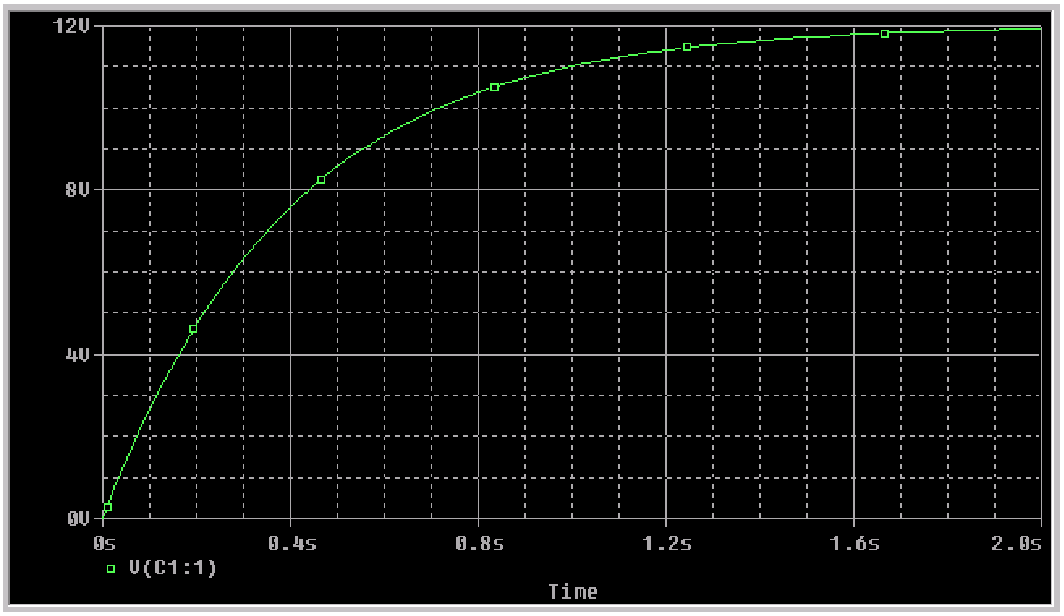

Figure 5

Figure 6 shows the voltage vs. time across the resistor using the program PSPICE.

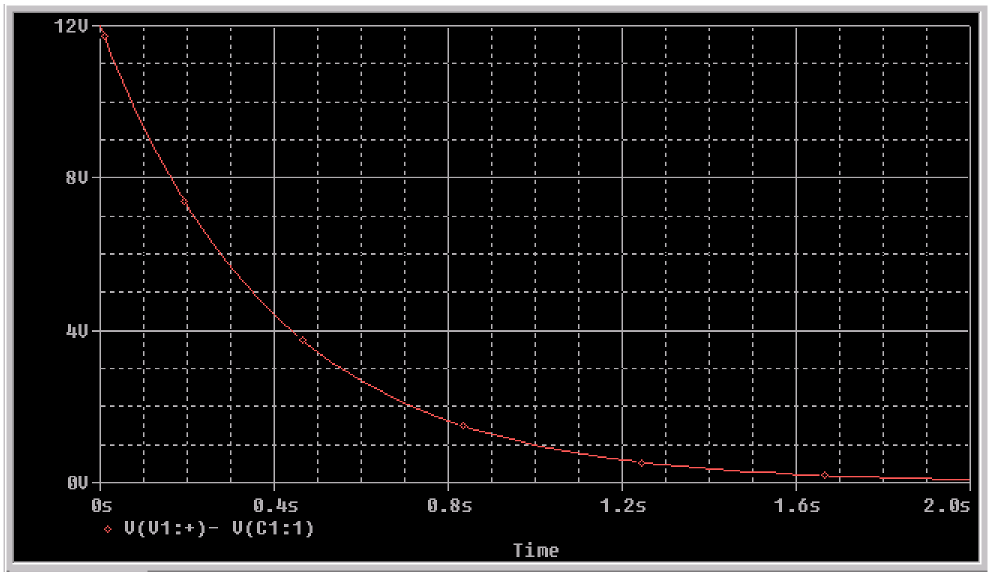

Figure 6

It is quite clear that the plots produced in the theoretical along with the plots produced in PSPICE indeed correspond to each other as expected. The voltage across the capacitor in the RC circuit asymptotically approaches 12V as time goes on. On the other hand, the voltage across the resistor in the RC circuit exponential decays to zero after about five time constants.

Steady-State Voltage Value of the RC Circuit:

The steady-state voltage value for the capacitor in the RC circuit happens when time approaches infinity, which is 12V. On the other hand, the steady-state voltage across the resistor is 0V as time approaches infinity. We can find the steady state values in the RC circuit by replacing the capacitor by an open circuit shown in Figure 7.

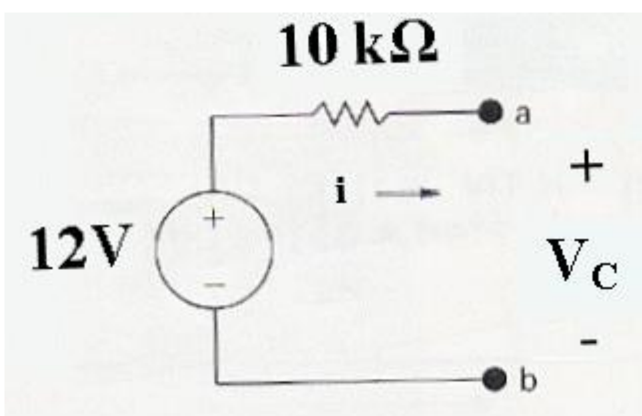

Figure 7

Since the current in the open circuit is zero, we can do a KVL around the loop in Figure 7 and find that the steady-state voltage value across the capacitor indeed is 12V, which can also be seen in Figure 2 and Figure 5 as time approaches infinity. Since the current across the resistor is 0A, the steady state voltage across the resistor is 0V, which is also depicted in Figure 3 and Figure 6 as time approaches infinity.

<u>Theoretical Analysis of a RL Circuit:</u>

The RL Circuit will contain a 12V dc independent voltage source, a resistor, and an inductor in series. We can find the step response of a first-order RL circuit by analyzing the circuit shown in Figure 8.

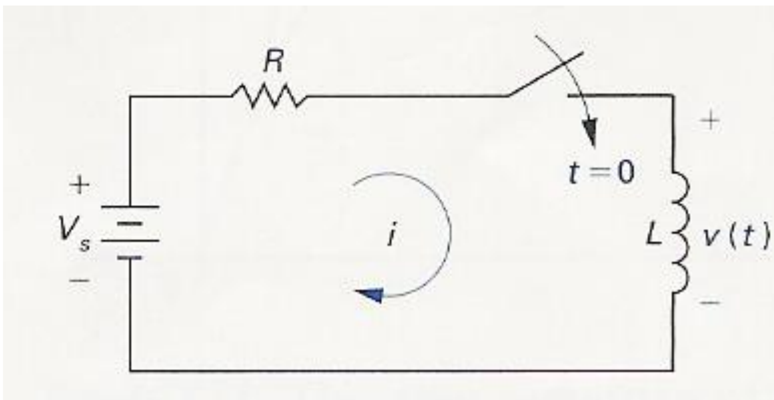

Figure 8

After the switch has been closed in the figure, we take a KVL around the loop which is

$$-V_S + Ri + v(t) = Ri + L\frac{di}{dt}$$

We can divide by L and rearrange

$$\frac{di}{dt} = -\frac{R}{L}\left(i - \frac{V_s}{R}\right)$$

Multiply by dt

$$di = -\frac{R}{L}\left(i - \frac{V_s}{R}\right)dt$$

Rearrange once again

$$\frac{di}{\left(i-\frac{V_S}{R}\right)}=-\frac{R}{L}$$

Integrate and put dummy variables in equation

$$\int_{i_o=I_o}^{i(t)}\frac{dx}{\left(x-\frac{V_S}{R}\right)}=-\frac{R}{L}\int_0^t dy$$

Perform the integration

$$\ln\left(\frac{i(t)-\frac{V_S}{R}}{I_o-\frac{V_S}{R}}\right)=-\frac{Rt}{L}$$

Take exponential of both sides and rearrange and solve for *i(t)*

$$i(t)=\frac{V_S}{R}+\left(I_o-\frac{V_S}{R}\right)\exp\left(-\frac{t}{\tau}\right) \quad , \text{ where } \quad \tau=\frac{L}{R}$$

When $I_o = 0$, the equation becomes

$$i(t)=\frac{V_S}{R}-\frac{V_S}{R}\exp\left(-\frac{t}{\tau}\right)$$

$$i(\infty)=\frac{V_S}{R} \qquad \text{and} \quad i(0)=\frac{V_S}{R}-\frac{V_S}{R}=0$$

After 1 time constant

$$i(\tau)=\frac{V_S}{R}-\frac{V_S}{R}\exp(-1)\approx 0.6321\frac{V_S}{R}$$

If the current were to continue to increase at its initial rate, it would reach its final value when $t = \tau$; because

$$\frac{di}{dt}=-\frac{V_S}{R}\left(\frac{-1}{\tau}\right)\exp\left(-\frac{t}{\tau}\right)=\frac{V_S}{L}\exp\left(-\frac{t}{\tau}\right)$$

The initial rate at which *i(t)* increases is

$$\frac{di}{dt}(0) = \frac{V_S}{L}$$

If the current were to continue to increase at this rate, the expression of *i* would be

$$i = \frac{V_S}{L}t$$

from which, at t = τ

$$i = \frac{V_S}{L}\frac{L}{R} = \frac{V_S}{R}$$

The voltage across an inductor is $L\frac{di}{dt}$, so for $t \geq 0^+$ the voltage across the inductor is

$$V_L(t) = L\left(-\frac{R}{L}\right)\left(I_o - \frac{V_S}{R}\right)\exp\left(-\frac{t}{\tau}\right) = (V_S - I_o R)\exp\left(-\frac{t}{\tau}\right) \qquad \text{equation \# 3}$$

The voltage across the resistor is

$$V_R(t) = V_S(t) - V_L(t)\ , \qquad \text{where } V_S(t) = I_s R \qquad \text{equation \# 4}$$

Now that the differential equation is solved, we can proceed to choose values. Since the time constant value has to be doubled for this part

$$\tau = 2 * 0.4\sec = 0.8\,\text{seconds}$$

We will decide to use the same resistor in the RC circuit of 10kΩ

Find the inductor value L

$$\tau = \frac{L}{R}$$

$$0.8 = \frac{L}{10{,}000}$$

L = 8,000 H

Plot until 5τ = 4 seconds

Use equation # 3 to find the voltage across the inductor

$$V_L(t) = (V_S - I_o R)\exp\left(-\frac{t}{\tau}\right) \quad , \quad I_o = 0$$

$$V_L(t) = (12V - 0)\exp(-1.25t)$$

$$V_L(t) = 12\exp(-1.25t)\ \text{V}$$

The plot for the voltage vs. time for the inductor can be seen in Figure 9.

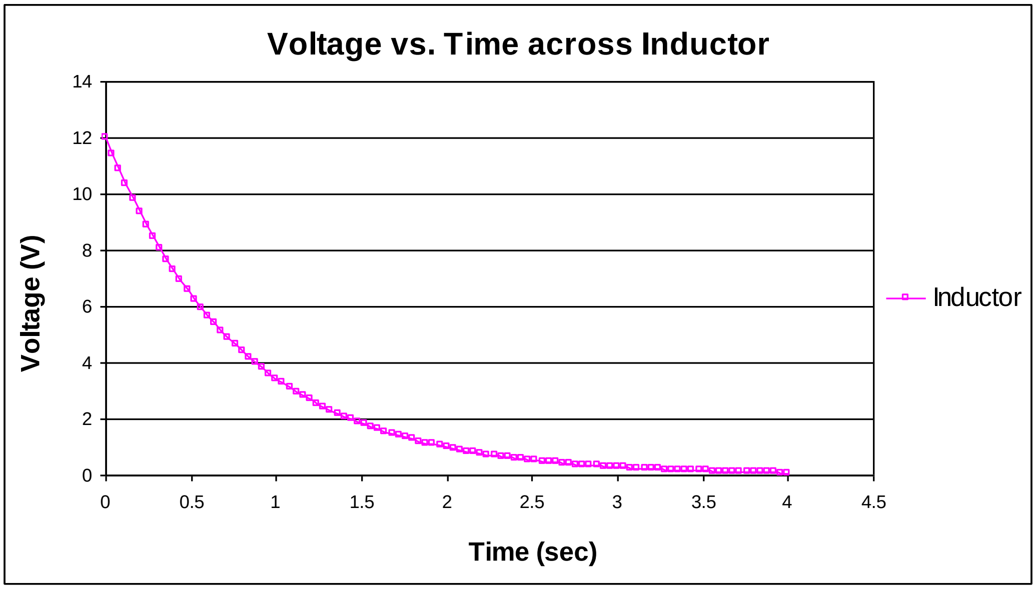

Figure 9

Use equation # 4 to find the voltage across the resistor

$V_R(t) = V_S(t) - V_L(t)$

$V_R(t)$ = 12V – [12exp(-1.25t) V]

$V_R(t)$ = 12 - 12exp(-1.25t) V

The plot for the voltage vs. time across the resistor can be seen in Figure 10.

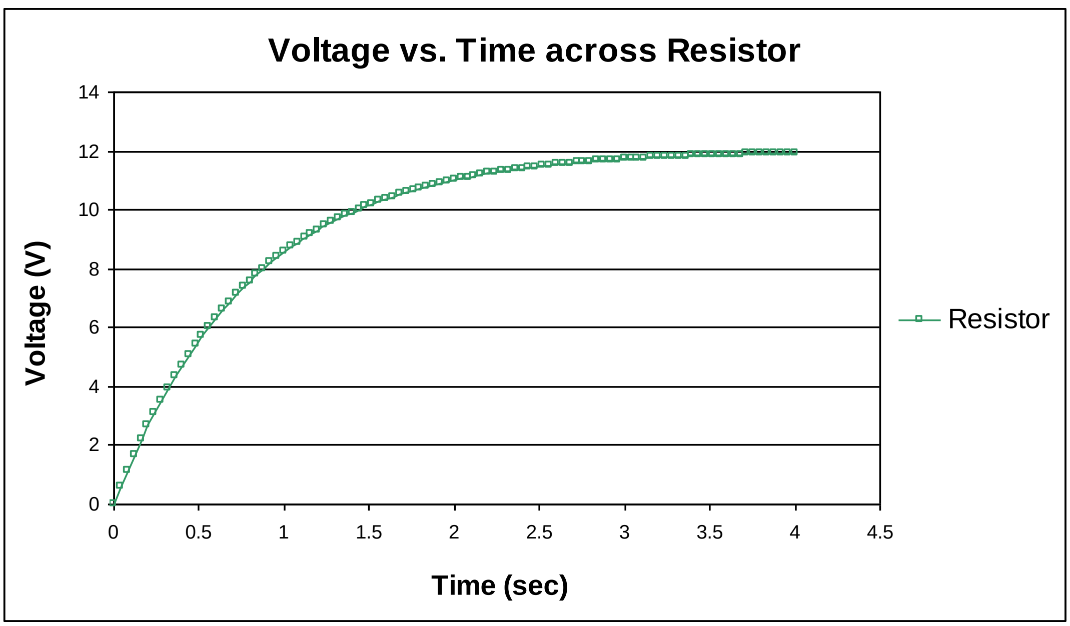

Figure 10

PSPICE Analysis of a RL Circuit:

For this particular part, we used the program PSPICE to perform the transient analysis. Figure 11 shows the circuit layout with the values we chose which would give us a time constant which would be twice as much for the RC circuit.

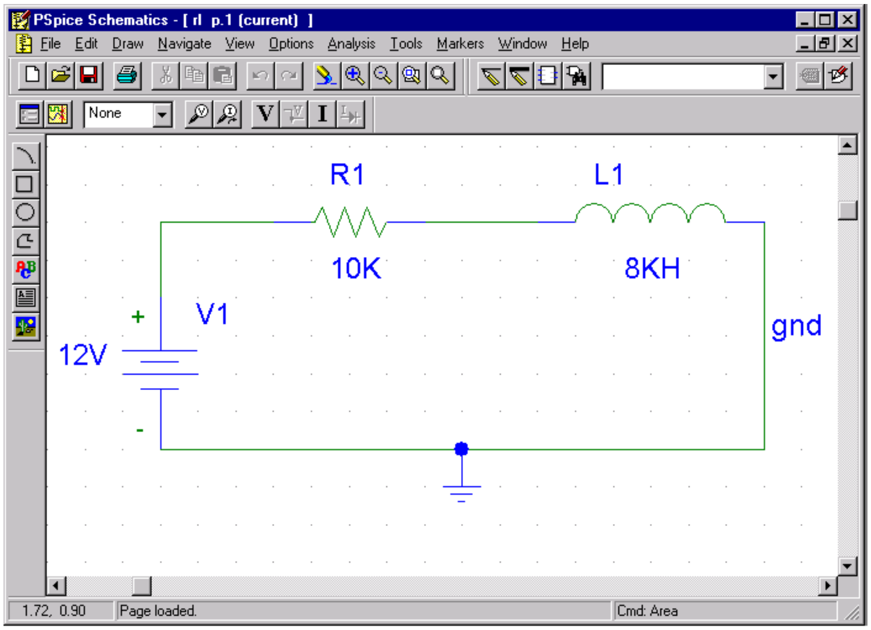

Figure 11

Since out final time will be 4 seconds, under the "Print Step" we set it equal to 0.04 seconds. The plot of voltage vs. time across the inductor using PSPICE can be seen in Figure 12.

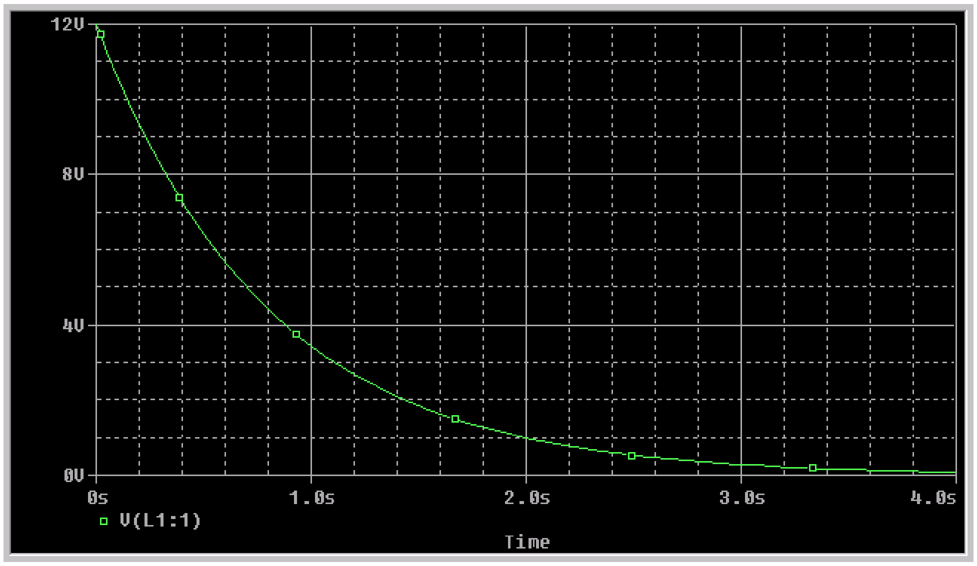

Figure 12

Figure 13 shows the voltage vs. time across the resistor using the program PSPICE.

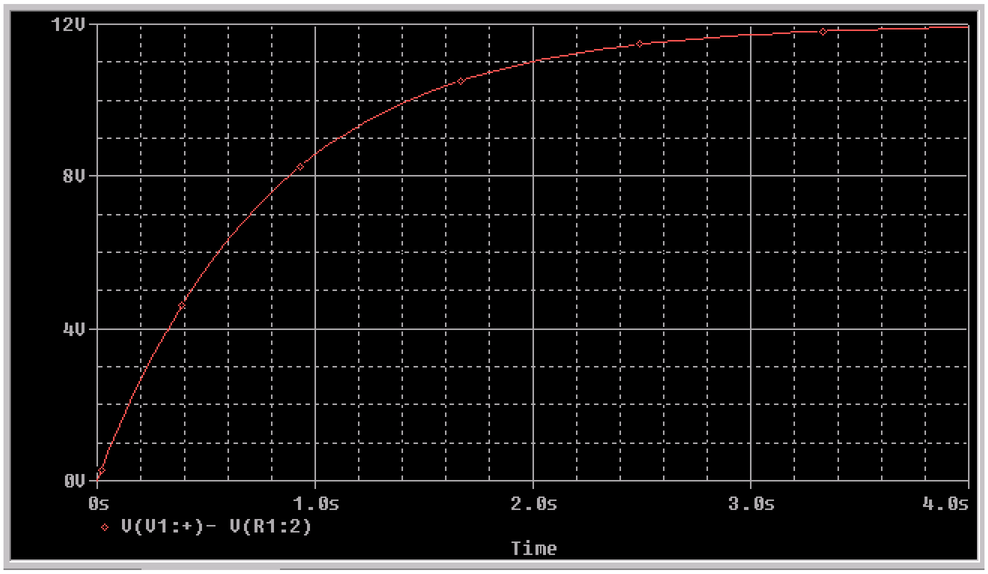

Figure 13

It is quite clear that the plots produced in the theoretical along with the plots produced in PSPICE indeed correspond to each other as expected. The voltage across the inductor in the RL circuit tends to exponential decay towards zero after about five time constants or 4 second. On the other hand, the voltage across the resistor in the RL circuit asymptotically approaches 12V as time goes on towards infinity.

Steady-State Voltage Value of the RL Circuit:

The steady-state voltage value for the inductor in the RL circuit happens when time approaches infinity, which is 0V. On the other hand, the steady-state voltage across the resistor is 12V as time approaches infinity. We can find the steady-state values in the RL circuit by replacing the inductor with a short circuit which can be seen in Figure 14.

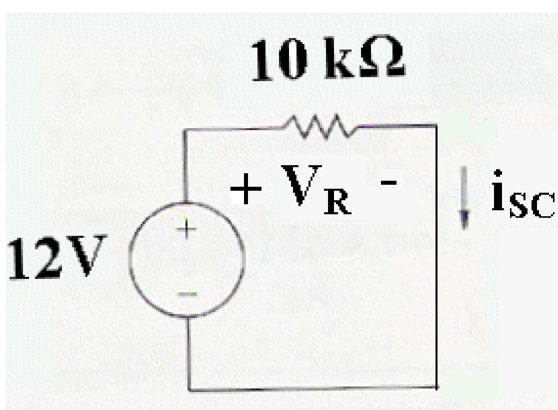

Figure 14

Since the voltage across the short circuit is 0V, we can therefore conclude that the steady-state voltage value across the inductor is 0V, which can also be seen in Figure 9 and Figure 12 as time approaches infinity. We then can do a KVL around the circuit in Figure 14 and find the value across the resistor which is 12V. Therefore the steady-state voltage value across the resistor is 12V, which is also depicted in Figure 10 and Figure 13 as time approaches infinity.